Solutions - Homework 1

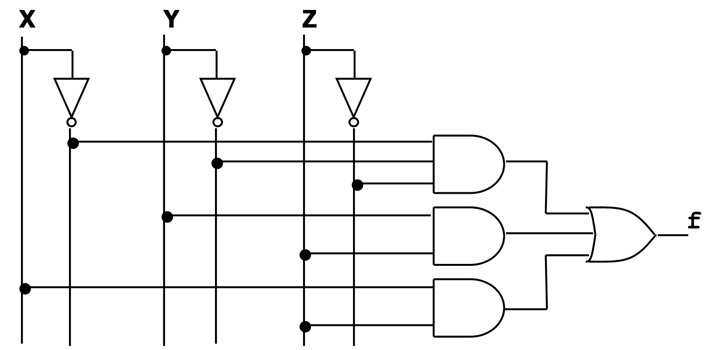
(Due date: September 15th @ 5:30 pm)
Presentation and clarity are very important!

PROBLEM 1 (25 PTS)

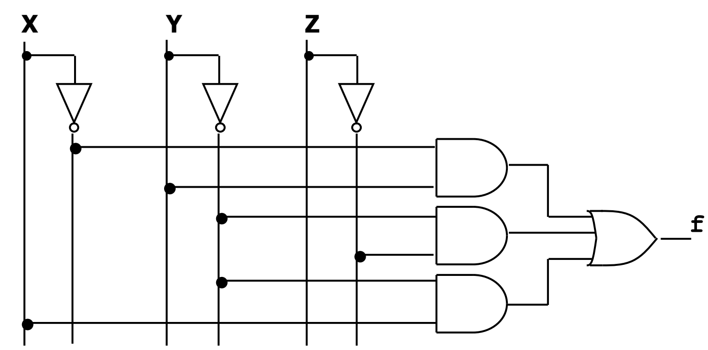
a) Simplify the following functions using ONLY Boolean Algebra Theorems. For each resulting simplified function, sketch the logic circuit using AND, OR, XOR, and NOT gates. (12 pts)

- ✓ $F(X,Y,Z) = \prod(M_1, M_2, M_4, M_6)$
- ✓ $F = \overline{(X\oplus\bar{Y})Z + XY\bar{Z}}$
- ✓ $F = (A+\bar{C}+\bar{D})(\bar{B}+\bar{C}+D)(A+\bar{B}+\bar{C})$
- ✓ $F = \overline{B(\bar{C}+\bar{A}) + \overline{AB}}$

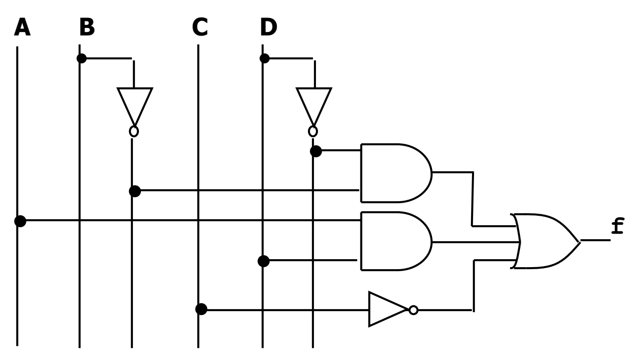
✓ $F(X,Y,Z) = \prod(M_1, M_2, M_4, M_6) = \sum(m_0, m_3, m_5, m_7) = \bar{X}\bar{Y}\bar{Z} + \bar{X}YZ + X\bar{Y}Z + XYZ = \bar{X}\bar{Y}\bar{Z} + Z(\bar{X}Y + X\bar{Y} + XY)$
$= \bar{X}\bar{Y}\bar{Z} + Z(\bar{X}Y + X) = \bar{X}\bar{Y}\bar{Z} + Z(\bar{X}+X)(Y+X) = \bar{X}\bar{Y}\bar{Z} + ZY + ZX$



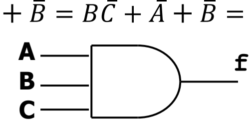
✓ $F = \overline{(X\oplus\bar{Y})Z + XY\bar{Z}} = \overline{(XY+\bar{X}\bar{Y})Z + XY\bar{Z}} = \overline{XYZ + \bar{X}\bar{Y}Z + XY\bar{Z}} = \overline{XY + \bar{X}\bar{Y}Z} = \overline{XY}.\overline{\bar{X}\bar{Y}Z}$
$= (\bar{X}+\bar{Y})(X+Y+\bar{Z}) = \bar{X}Y + \bar{X}\bar{Z} + \bar{Y}X + \bar{Y}\bar{Z} = Y\bar{X} + \bar{Y}\bar{Z} + \bar{X}\bar{Z} + \bar{Y}X = Y\bar{X} + \bar{Y}\bar{Z} + \bar{Y}X$



✓ $F = (A+\bar{C}+\bar{D})(\bar{B}+\bar{C}+D)(A+\bar{B}+\bar{C}) = (D+\bar{B}+\bar{C})(\bar{D}+A+\bar{C})(A+\bar{B}+\bar{C}) = (D+\bar{B}+\bar{C})(\bar{D}+A+\bar{C})$
$= (D+\bar{B}+\bar{C})(\bar{D}+A+\bar{C}) = D(A+\bar{C}) + \bar{D}(\bar{B}+\bar{C}) + (A+\bar{C})(\bar{B}+\bar{C}) = D(A+\bar{C}) + \bar{D}(\bar{B}+\bar{C})$
$= \bar{D}\bar{B} + DA + \bar{C}$



✓ $F = \overline{B(\bar{C}+\bar{A}) + \overline{AB}} = \overline{B\bar{C} + B\bar{A} + \bar{A} + \bar{B}} = \overline{B\bar{C} + \bar{A} + \bar{B}} = \overline{\bar{A} + (\bar{B}+B)(\bar{B}+\bar{C})} = \overline{\bar{A}+\bar{B}+\bar{C}} = ABC$



b) Using ONLY Boolean Algebra Theorems, determine whether or not the following expression is valid, i.e., whether the left- and right-hand sides represent the same function: (5 pts)

$$x_1\overline{x_3} + x_2x_3 + \overline{x_2}\,\overline{x_3} = (x_1+\overline{x_2}+x_3)(x_1+x_2+\overline{x_3})(\overline{x_1}+x_2+\overline{x_3})$$

$$(x_1+\overline{x_2}+x_3)(x_1+x_2+\overline{x_3})(\overline{x_1}+x_2+\overline{x_3}) = \left(x_1 + (\overline{x_2}+x_3)(x_2+\overline{x_3})\right)(\overline{x_1}+x_2+\overline{x_3}) = (x_1+\overline{x_2}\,\overline{x_3}+x_2x_3)(\overline{x_1}+x_2+\overline{x_3})$$
$$= x_1x_2 + x_1\overline{x_3} + \overline{x_2}\,\overline{x_3}\,\overline{x_1} + \overline{x_2}\,\overline{x_3} + x_2x_3\overline{x_1} + x_2x_3 = x_1x_2 + x_1\overline{x_3} + \overline{x_2}\,\overline{x_3} + x_2x_3 = x_3x_2 + \overline{x_3}x_1 + x_2x_1 + \overline{x_2}\,\overline{x_3}$$
$$= x_3x_2 + \overline{x_3}x_1 + \overline{x_2}\,\overline{x_3} = x_1\overline{x_3} + x_2x_3 + \overline{x_2}\,\overline{x_3}$$

c) For the following Truth table with two outputs: (8 pts)
- Provide the Boolean functions using the Canonical Sum of Products (SOP), and Product of Sums (POS).
- Express the Boolean functions using the minterms and maxterms representations.
- Sketch the logic circuits as Canonical Sum of Products and Product of Sums.

x	y	z	f_1	f_2
0	0	0	0	1
0	0	1	0	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	1	1
1	1	1	1	0

Sum of Products

$f_1 = \bar{X}Y\bar{Z} + X\bar{Y}\bar{Z} + XY\bar{Z} + XYZ$

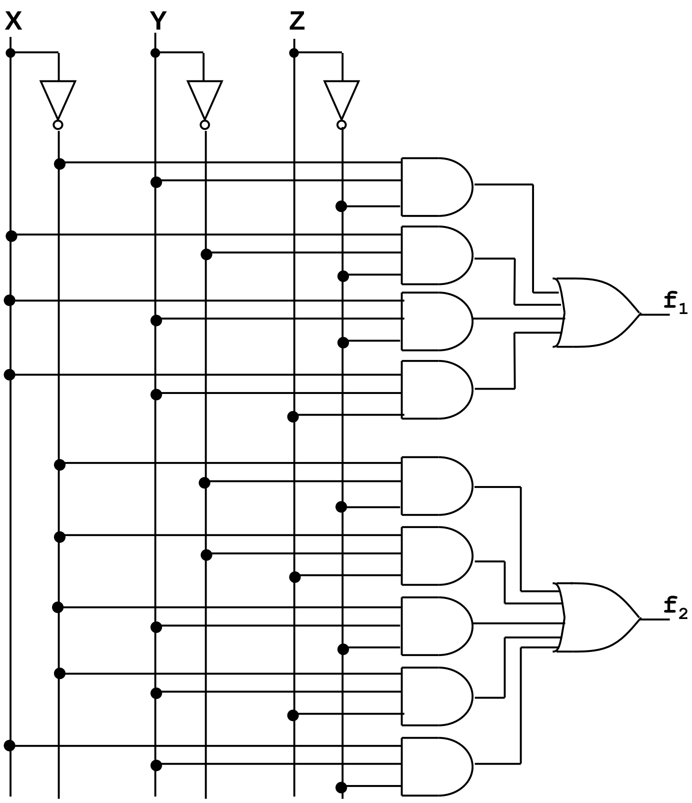
$f_2 = \bar{X}\bar{Y}\bar{Z} + \bar{X}\bar{Y}Z + \bar{X}Y\bar{Z} + \bar{X}YZ + XY\bar{Z}$

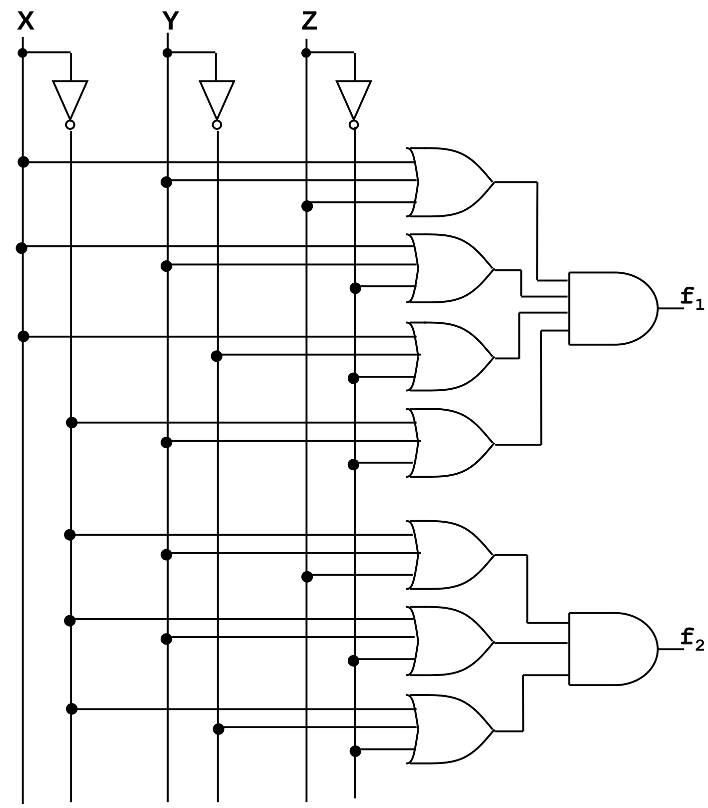
Product of Sums

$f_1 = (X+Y+Z)(X+Y+\bar{Z})(X+\bar{Y}+\bar{Z})(\bar{X}+Y+\bar{Z})$

$f_2 = (\bar{X}+Y+Z)(\bar{X}+Y+\bar{Z})(\bar{X}+\bar{Y}+\bar{Z})$

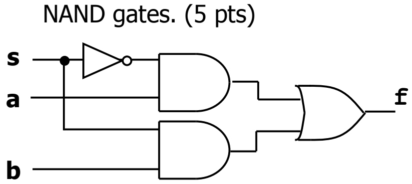
Minterms and maxterms: $f_1 = \sum(m_2, m_4, m_6, m_7) = \prod(M_0, M_1, M_3, M_5)$.
$f_2 = \sum(m_0, m_1, m_2, m_3, m_6) = \prod(M_4, M_5, M_7)$.





PROBLEM 2 (15 PTS)

a) The following circuit has the following logic function: $f = \bar{s}a + sb$.
- Complete the truth table of the circuit, and sketch the logic circuit using ONLY 2-input NAND gates. (5 pts)



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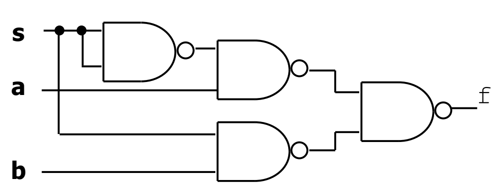
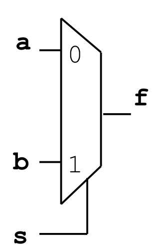
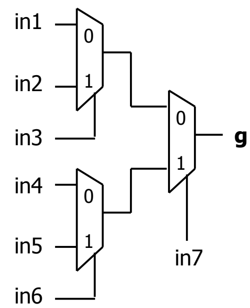




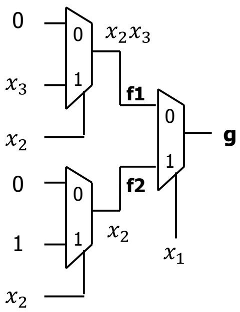
s	a	b	f
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

b) The circuit on the right can be used to realize various different functions. (10 pts)



- For example, the following selection of inputs produce the function: $g = x_1x_2 + x_2x_3$. Demonstrate that this is the case.

in1	in2	in3	in4	in5	in6	in7
0	x_3	x_2	0	1	x_2	x_1

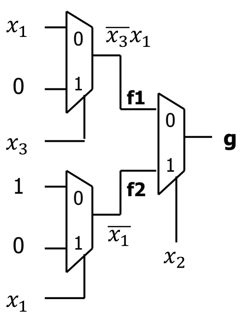


$f_1 = \overline{x_2}(0) + x_2(x_3) = x_2x_3$
$f_2 = \overline{x_2}(0) + x_2(1) = x_2$

$g = \overline{x_1}(x_2x_3) + x_1(x_2) = x_2(x_1 + \overline{x_1}x_3) = x_2(x_1 + \overline{x_1})(x_1 + x_3)$
$g = x_2(x_1 + x_3) = x_1x_2 + x_2x_3$

- Given the following inputs, provide the resulting function g (minimize the function).

in1	in2	in3	in4	in5	in6	in7
x_1	0	x_3	1	0	x_1	x_2

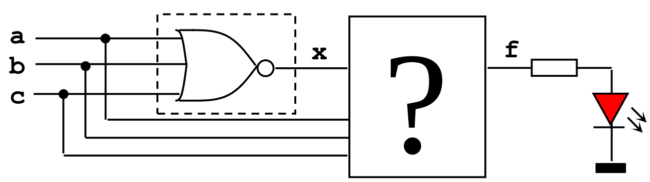


$f_1 = \overline{x_3}(x_1) + x_3(0) = \overline{x_3}(x_1)$
$f_2 = \overline{x_1}(1) + x_3(0) = \overline{x_1}$

$g = \overline{x_2}(\overline{x_3}x_1) + x_2(\overline{x_1}) = \overline{x_1}x_2 + \overline{x_2}x_1\overline{x_3}$

PROBLEM 3 (12 PTS)

- Design a circuit (<u>simplify your circuit</u>) that verifies the logical operation of a 3-input NOR gate. f = '1' (LED ON) if the NOR gate does NOT work properly. Assumption: when the NOR gate is not working, it generates 1's instead of 0's and vice versa.



x	a	b	c	f	x_{good}
0	0	0	0	1	1
0	0	0	1	0	0
0	0	1	0	0	0
0	0	1	1	0	0
0	1	0	0	0	0
0	1	0	1	0	0
0	1	1	0	0	0
0	1	1	1	0	0
1	0	0	0	0	1
1	0	0	1	1	0
1	0	1	0	1	0
1	0	1	1	1	0
1	1	0	0	1	0
1	1	0	1	1	0
1	1	1	0	1	0
1	1	1	1	1	0

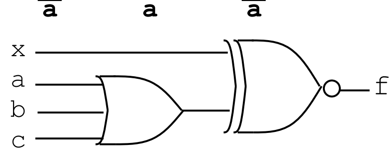
bc \ xa	00	01	11	10
00	1	0	1	0
01	0	0	1	1
11	0	0	1	1
10	0	0	1	1

$f = xa + xb + xc + \bar{x}\bar{a}\bar{b}\bar{c}$

$f = \bar{x}\bar{a}\bar{b}\bar{c} + x(a + b + c)$

$f = \bar{x}(\overline{a + b + c}) + x(a + b + c)$

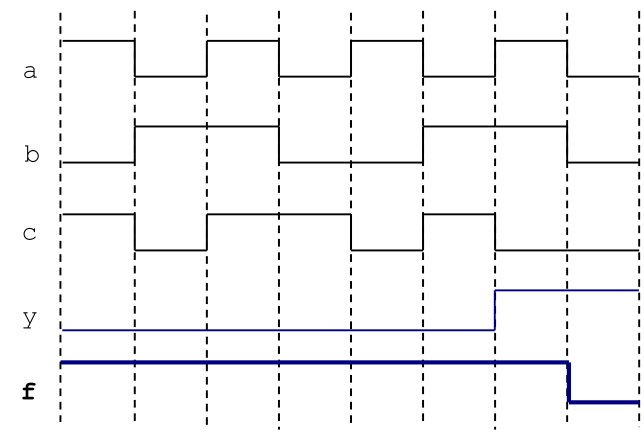
$f = \overline{x \oplus (a + b + c)}$



PROBLEM 4 (20 PTS)

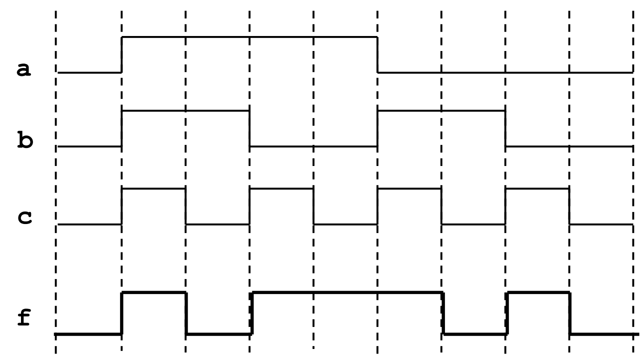
a) Complete the timing diagram of the logic circuit whose VHDL description is shown below: (6 pts)

```
library ieee;
use ieee.std_logic_1164.all;

entity circ is
  port ( a, b, c: in std_logic;
         f: out std_logic);
end circ;

architecture st of circ is
  signal x, y: std_logic;

begin
  x <= a xor b;
  y <= x nor c;
  f <= y nand (not b);
end st;
```



b) The following is the timing diagram of a logic circuit with 3 inputs. Sketch the logic circuit that generates this waveform. Then, complete the VHDL code. (8 pts)

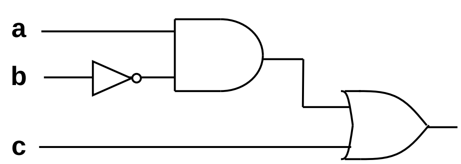
```
library ieee;
use ieee.std_logic_1164.all;

entity wav is
  port ( a, b, c: in std_logic;
               f: out std_logic);
end wav;

architecture st of wav is

begin
   f <= c or (a and not (b));
end st;
```



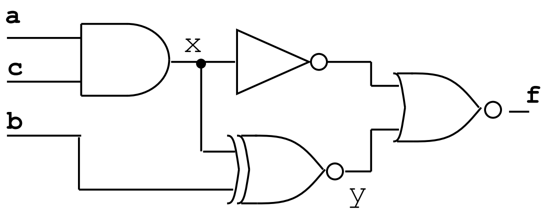
a	b	c	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

c \ ab	00	01	11	10
0	0	0	0	1
1	1	1	1	1

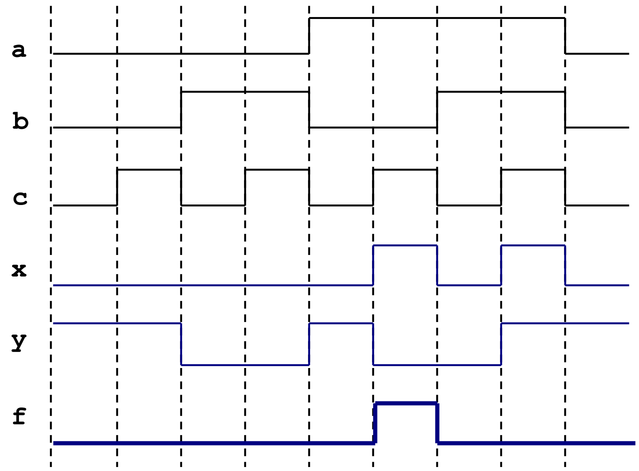
$f = c + a\bar{b}$



c) Complete the timing diagram of the following circuit: (6 pts)



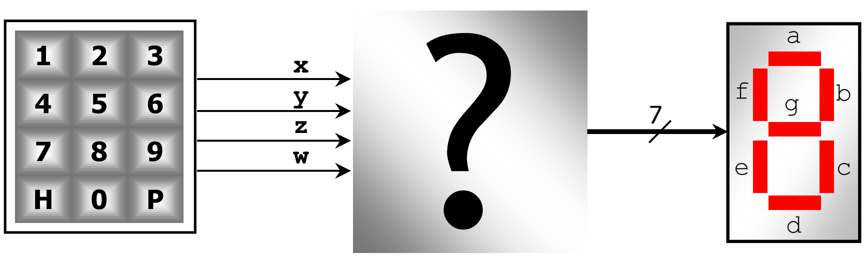
$f = a\bar{b}c$



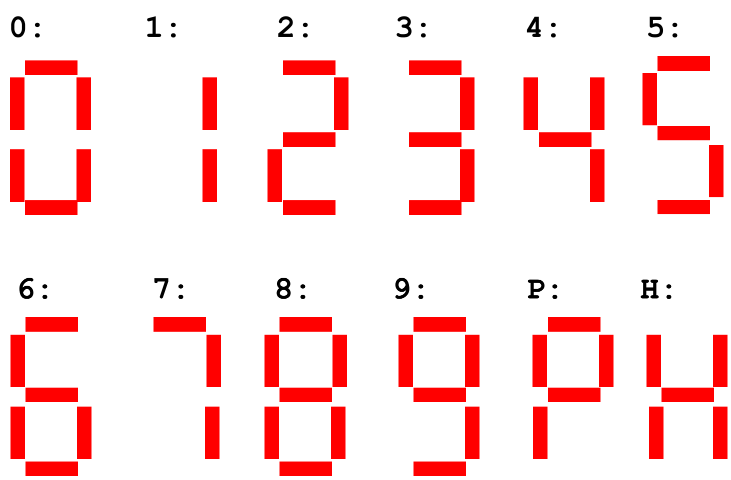
PROBLEM 5 (28 PTS)

- A numeric keypad produces a 4-bit code as shown below. We want to design a logic circuit that converts each 4-bit code to a 7-segment code, where each segment is an LED: A LED is ON if it is given a logic '1'. A LED is OFF if it is given a logic '0'.

✓ Complete the truth table for each output (a, b, c, d, e, f, g).
✓ Provide the simplified expression for each output (a, b, c, d, e, f, g). Use Karnaugh maps for c, d, e, f, g and the Quine-McCluskey algorithm for a, b. Note that it is safe to assume that the codes 1100 to 1111 will not be produced by the keypad.



Value	X	Y	Z	W	a	b	c	d	e	f	g
0	0	0	0	0							
1	0	0	0	1							
2	0	0	1	0							
3	0	0	1	1							
4	0	1	0	0							
5	0	1	0	1							
6	0	1	1	0							
7	0	1	1	1							
8	1	0	0	0							
9	1	0	0	1	1	1	1	1	0	1	1
P	1	0	1	0							
H	1	0	1	1							
	1	1	0	0							
	1	1	0	1							
	1	1	1	0							
	1	1	1	1							



Value	X	Y	Z	W	a	b	c	d	e	f	g
0	0	0	0	0	1	1	1	1	1	1	0
1	0	0	0	1	0	1	1	0	0	0	0
2	0	0	1	0	1	1	0	1	1	0	1
3	0	0	1	1	1	1	1	1	0	0	1
4	0	1	0	0	0	1	1	0	0	1	1
5	0	1	0	1	1	0	1	1	0	1	1
6	0	1	1	0	1	0	1	1	1	1	1
7	0	1	1	1	1	1	1	0	0	0	0
8	1	0	0	0	1	1	1	1	1	1	1
9	1	0	0	1	1	1	1	1	0	1	1
P	1	0	1	0	1	1	0	0	1	1	1
H	1	0	1	1	0	1	1	0	1	1	1
	1	1	0	0	X	X	X	X	X	X	X
	1	1	0	1	X	X	X	X	X	X	X
	1	1	1	0	X	X	X	X	X	X	X
	1	1	1	1	X	X	X	X	X	X	X

c

zw \ xy	00	01	11	10
00	1	1	X	1
01	1	1	X	1
11	1	1	X	1
10	0	1	X	0

d

zw \ xy	00	01	11	10
00	1	0	X	1
01	0	1	X	1
11	1	0	X	0
10	1	1	X	0

e

zw \ xy	00	01	11	10
00	1	0	X	1
01	0	0	X	0
11	0	0	X	1
10	1	1	X	1

f

zw \ xy	00	01	11	10
00	1	1	X	1
01	0	1	X	1
11	0	0	X	1
10	0	1	X	1

$c = y + \bar{z} + w$
$d = x\bar{z} + \bar{x}\bar{y}\bar{w} + \bar{x}\bar{y}z + \bar{z}wy + z\bar{w}y$
$e = \bar{w}\bar{y} + z\bar{w} + xz$
$f = x + \bar{z}\bar{w} + y\bar{z} + y\bar{w}$
$g = x + z\bar{w} + y\bar{z} + \bar{y}z$

g

zw \ xy	00	01	11	10
00	0	1	X	1
01	0	1	X	1
11	1	0	X	1
10	1	1	X	1

- $a = \sum m(0,2,3,5,6,7,8,9,10) + \sum d(12,13,14,15)$.
  Too many minterms. We better optimize: $\bar{a} = \sum m(1,4,11) + \sum d(12,13,14,15)$

Number of ones	4-literal implicants	3-literal implicants	2-literal implicants	1-literal implicants
1	**m_1 = 0001** m_4 = 0100 ✓	**$m_{4,12}$ = -100**		
2	m_{12}= 1100 ✓	$m_{12,13}$ = 110- ✓ $m_{12,14}$ = 11-0 ✓	**$m_{12,13,14,15}$ = 11--** ~~$m_{12,14,13,15}$ = 11--~~ ✓	
3	m_{11}= 1011 ✓ m_{13}= 1101 ✓ m_{14}= 1110 ✓	$m_{13,15}$ = 11-1 ✓ $m_{14,15}$ = 111- ✓ **$m_{11,15}$ = 1-11**		
4	m_{15}= 1111 ✓			

$\bar{a} = \bar{x}\bar{y}\bar{z}w + y\bar{z}\bar{w} + xzw + xy$

Prime Implicants		Minterms		
		1	4	11
m_1	$\bar{x}\bar{y}\bar{z}w$	**X**		
$m_{4,12}$	$y\bar{z}\bar{w}$		**X**	
$m_{11,15}$	xzw			**X**
$m_{12,13,14,15}$	xy			

$\bar{a} = \bar{x}\bar{y}\bar{z}w + y\bar{z}\bar{w} + xzw \Rightarrow \qquad a = (x + y + z + \bar{w})(\bar{y} + z + w)(\bar{x} + \bar{z} + \bar{w})$

- $b = \sum m(0,1,2,3,4,7,8,9,10,11) + \sum d(12,13,14,15)$.
  Too many minterms. We better optimize: $\bar{b} = \sum m(5,6) + \sum d(12,13,14,15)$

Number of ones	4-literal implicants	3-literal implicants	2-literal implicants	1-literal implicants
2	m_5 = 0101 ✓ m_6 = 0110 ✓ m_{12}= 1100 ✓	**$m_{5,13}$ = -101** **$m_{6,14}$ = -110** $m_{12,13}$ = 110- ✓ $m_{12,14}$ = 11-0 ✓	**$m_{12,13,14,15}$ = 11--** ~~$m_{12,14,13,15}$ = 11-- ✓~~	
3	m_{13}= 1101 ✓ m_{14}= 1110 ✓	$m_{13,15}$ = 11-1 ✓ $m_{14,15}$ = 111- ✓		
4	m_{15}= 1111 ✓			

$\bar{b} = \bar{x}\bar{y}\bar{z}w + y\bar{z}\bar{w} + xzw + xy$

Prime Implicants		Minterms 5	Minterms 6
$m_{5,13}$	$y\bar{z}w$	**X**	
$m_{6,14}$	$yz\bar{w}$		**X**
$m_{12,13,14,15}$	xy		

$\bar{b} = y\bar{z}w + yz\bar{w} \quad \Rightarrow \quad b = (\bar{y} + z + \bar{w})(\bar{y} + \bar{z} + w)$